Payment term.

He gives up the reversion to £m, payable at his decease, the present value of which, multiplied by D_{x+n}, is . . . . $\Big\}\; m.M_{x+n}$.

$$\text{Equating, } m'.M_{x+n}+P_x.N_{x+n-1}=m.M_{x+n};$$

$$\text{whence } m'=\frac{m.M_{x+n}-P_x.N_{x+n-1}}{M_{x+n}}=m-\frac{P_x.N_{x+n-1}}{M_{x+n}}.$$

$$\text{Let } m=1, \text{ then } m'=1-\frac{P_x.N_{x+n-1}}{M_{x+n}};$$

dividing by N_{x+n-1}, we have $\dfrac{P_x}{M_{x+n}\div N_{x+n-1}}$ and $\dfrac{M_{x+n}}{N_{x+n-1}}=P_{x+n}$;

$$\therefore\ m=1-\frac{P_x}{P_{x+n}},$$

which agrees with the expression given by Mr. Sprague (*Assurance Magazine*, vol. vii., p. 59), derived from the assurance and annuity values.

It is to be observed, that the foregoing Problems include only the case where the premium is just due, and which is the one that will generally occur in practice. For the case where the premium has just been paid, the formulæ will have to be modified by the omission of -1 from all the terms of column N.

FORMULÆ FOR THE ANNUAL PREMIUM FOR A TERM ASSURANCE ON TWO JOINT LIVES.

To the Editor of the Assurance Magazine.

SIR,—It is probably very seldom that an assurance is effected for a term of years on the joint duration of two lives; and when such a case occurs, it will often be thought desirable to employ some method of approximation in order to determine the proper premium for the assurance. One such method may be mentioned. If a policy is to be effected for t years, on the joint duration of two lives, let it be calculated what would be the surrender value of a policy for the whole duration of the same lives, after it has been t years in force, or what percentage of the premiums paid would be returned; then if that percentage be deducted from the Office premium for an assurance for the whole duration of the lives, the remainder will be the premium for the term assurance. But it is desirable to show how such a premium may be calculated exactly, and the suitable formulæ will probably be new to many of the readers of the *Assurance Magazine*, as they are not given in David Jones's *Treatise on Annuities*, nor in any other work of which I am aware.

Let (a) denote the value of an annuity of £1 on the joint lives of the last v survivors of the lives m, m_1, m_2, &c.; and (A) the value of an assurance of £1 on the same lives. Then, it is proved by Jones (Art. 197), that

$$(A)_{\overline{t}|}=r\{1+(a)_{\overline{t-1}|}\}-(a)_{\overline{t}|}.$$

Also, let (P) denote the annual premium for the same assurance;

then,
$$(\mathrm{P})_{\overline{t}|}=\frac{(\mathrm{A})_{\overline{t}|}}{1+(a)_{\overline{t-1}|}};$$

or, substituting the value of $(\mathrm{A})_{\overline{t}|}$ given above,
$$(\mathrm{P})_{\overline{t}|}=r-\frac{(a)_{\overline{t}|}}{1+(a)_{\overline{t-1}|}}.$$

which is a general formula for the premium for a term assurance.

When there is only one life, aged m, we have
$$a_{m\overline{t}|}=\frac{\mathrm{N}_m-\mathrm{N}_{m+t}}{\mathrm{D}_m},\qquad 1+a_{m\overline{t-1}|}=\frac{\mathrm{N}_{m-1}-\mathrm{N}_{m+t-1}}{\mathrm{D}_m};$$

and therefore,
$$\mathrm{P}_{m\overline{t}|}=r-\frac{\mathrm{N}_m-\mathrm{N}_{m+t}}{\mathrm{N}_{m-1}-\mathrm{N}_{m+t-1}};$$

which is the formula given by Jones in Art. 200. When there are two lives, m, n, the formula is perfectly analogous.

Thus,
$$a_{m.n\overline{t}|}=\frac{\mathrm{N}_{m.n}-\mathrm{N}_{m+t.n+t}}{\mathrm{D}_{m.n}},$$
$$1+a_{m.n\overline{t-1}|}=\frac{\mathrm{N}_{m-1.n-1}-\mathrm{N}_{m+t-1.n+t-1}}{\mathrm{D}_{m.n}};$$

whence,
$$\mathrm{P}_{m.n\overline{t}|}=r-\frac{\mathrm{N}_{m.n}-\mathrm{N}_{m+t.n+t}}{\mathrm{N}_{m-1.n-1}-\mathrm{N}_{m+t-1.n+t-1}}.$$

The calculation of the premium by this formula presents no more difficulty than the corresponding one for a single life. Of course, in the case of a single life, it is preferable to use the formula,
$$\mathrm{P}_{m\overline{t}|}=\frac{\mathrm{M}_m-\mathrm{M}_{m+t}}{\mathrm{N}_{m-1}-\mathrm{N}_{m+t-1}},$$

when the column M is formed. This formula is seen to be identical with the former by means of the relation $\mathrm{M}_m=r\mathrm{N}_{m-1}-\mathrm{N}_m$. The analogous formula for two joint lives is not available, because the column M is seldom or never calculated for two lives.

It is easily seen that the analogous formulæ for the annual premium for an assurance for the whole term of a single life, or of two joint lives, are—
$$\mathrm{P}_m=r-\frac{\mathrm{N}_m}{\mathrm{N}_{m-1}},\qquad \mathrm{P}_{m.n}=r-\frac{\mathrm{N}_{m.n}}{\mathrm{N}_{m-1.n-1}}.$$

These however will never be required in practice, since Orchard's Tables afford a simpler way of obtaining the annual premiums, viz., by deducing them from the tabulated values of the annuities.

In conclusion, I may remark, that it is a little singular Jones has not adapted the formulæ for joint life annuities and assurances to the Commutation (or D and N) Tables, while he has done so with the much more complicated formulæ for survivorship assurances.

I remain, Sir,

Your obedient Servant,

T. B. SPRAGUE.

Liverpool and London Insurance Company,
20, *Poultry, London,*
7th September, 1858.